Emilie Du Châtelet, *Foundations of Physics*, 1740.
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Footnotes are ours except where otherwise indicated.
Du Châtelet's marginal notes are placed in **{bold}** in the closest appropriate place in the text.
Please see the French original for the position of each note in the margin alongside the paragraph.
Figures are available in the original text, and online via the BNF.

Chapter 12. Of composite motion

§271. {Definition of composite motion} Composite motion is that in which the Body obeys several forces at once, which impress different directions on it, and which make it tend toward diverse points at the same time.

§272. {Some differences that the directions of the forces which impel a Body give to its motion} The motion of a Body which is impelled by two forces at the same time, is different according to how the action of these forces is directed:
1. If these forces act in the same direction, the moveable body[2] moves faster; but, the direction of its movement not being changed, this Body moves with a simple motion.
2. If these two forces are equal and opposed to one another, they cancel each other out.[3] Thus, the Body does not leave its place, and there is not any motion produced.
3. If the opposing forces are unequal, they cancel each other out only in part, and the resulting motion is the effect of the remainder of these two forces.
4. **{Fig. 19, Plate 4}** If these two forces are perpendicular to one another other, as, for example, the force designated by Line AB and the active force designated by Line AD, they will neither cancel each other out nor accelerate. Each will act upon the Body as if it was at rest; so the path of the moveable body will be changed, and this Body will have a motion composed of the motion impressed by these two forces.

This will be the case only when the two forces that act upon the Body are perpendicular to one another; in this case each acts on the Body as if it was at rest.
5. **{Plate 4, Fig. 20 and 21}** Finally, if these two forces are oblique to one another, such as the force AF and the force AE, or even such as the force AG and the force AH, they will retard or accelerate the motion of one another, according to how the obliquity of the Lines that represent them will be directed, and they will have moreover an action that is perpendicular to one another, that will neither accelerate

[1] Penelope Brading, Ashton Green, John Hanson, Lauren LaMore, Anne Seul, Phillip Sloan, Monica Solomon.
[2] We translate "mobile" as "moveable body" throughout.
[3] We have translated "elles se détruisent mutuellement" as "they cancel each other out" throughout. However, this might also be translated as "they destroy one another" or "they mutually destroy themselves".

nor retard the motion of one another.

§273. {Fig 22} If Body A is moved by any force in the direction AB and with the speed designated by this Line AB, and if the Body is impelled at the same time by a second force that impresses upon it the direction and the speed AC, then this Body, being moved by two forces that at the same time tend to make it traverse the two Lines AB and AC, will obey these two forces according to the quantity of their action upon it; and this Body will have a motion whose direction and speed will be composed from the speed and direction of the two forces that act upon it.

§274. In order to determine which line a Body that is thus moved will describe in its motion, let us imagine that the lines AC and AB are each divided into equal parts **{Fig. 22}** A, e, g, i, o, C, and A, F, H, K, M, B, and let us suppose that while the moveable body A traverses the divisions of the line AC, this line flows [coule] parallel to itself the length of the line AB, such that in the same time during which the Body A traverses the space Ae along the line AC, the line AC traverses space AF along the line AB. It is certain that at the end of this first moment, the moveable body will be found at point E. Similarly, if in the second instant, during which the moveable body goes from e to g along the line AC, this line flows from F to H **{Fig. 22**} along the line AB, then the moveable body at the end of this second instant will be at G. For the same reason, it be at I at the end of the third instant, then at O in the fourth, and finally at D in the fifth. {**A Body moved by two forces traverses the diagonal of a parallelogram**} Thus, if we draw the lines CD and BD parallel to AB and AC, and we thus obtain the parallelogram ABCD, the Body, in obeying the two forces AB and AC that act upon it at the same time, will describe the diagonal AD of this parallelogram; for the force that impels it towards AB has the same effect on it as the motion by which I supposed that the line AC traversed the line AB.

The quantity of transportation [quantité du transport] of the Body towards the line BD is therefore the effect of the force that acts from A towards B, and the quantity of its transportation towards the line CD is the effect of the force that acts from A towards C. Thus, these forces remain distinct in their composite effect.

§275. The moveable body traverses this diagonal AD in the same time in which it would have traversed the lines AC and AB separately. For, by the single force directed towards AB the Body will reach the line BD in the same time, whether or not the force towards AC is impressed upon it. Similarly, it will reach the line CD in the same time, by the force which directs it towards AC, whether or not the force towards AB is impressed upon it. **{Fig. 22}** Therefore, when the line AC, which I have supposed to flow along the line AB, will have arrived at BD, the Body A that traverses this line AC will then be at point C of this line AC. But the points C and D will then coincide. Thus, every Body that is moved by two powers that make between them any angle whatsoever, traverses the diagonal of the parallelogram formed by these lines, in which the length and the position represent the direction and the speed of these two forces. This diagonal represents the speed of the composite motion, and it is the resultant of the motions impressed on the moveable body.

§276. It follows from this that the motion of a Body can always be resolved into two other motions, by making the line along which a Body is moving become the diagonal of a parallelogram of which the two sides, in their length and their position, will represent the directions and the speeds of the two motions, into which that of the Body under consideration will be resolved.

§277. {**Fig. 23**} The angle EAB that the lines AB and AE, which mark the directions of the forces, make between themselves, is called the angle of direction.

§278. The line traversed by a Body impelled by two forces at the same time, is greater or lesser in length according to the angle of direction of the impelling forces. For, given that the lines AE and AB are equal in Figures 24, 25 and 26 {**Figs. 24, 25, 26**}, we easily see that the line AD, which is the path that the moveable body traverses in the same time, is not equal in these three Figures.

§279. The more acute the angle of direction EAB, as in Figure 24, the longer the line AD that the Body traverses; and the more obtuse the angle EAB, as in figure 25, the shorter the path of the moveable Body. For, in the first case **{Fig. 24}** the force that impels the Body along the line AE, and that can be resolved into the lines Af and Ag, combines [conspirer] with the force that impels the Body towards AB and augments it by the quantity Ag, or by its action perpendicular to Af. And in the second case **{Fig. 25}** the force that impels the Body toward AE, decomposed as in the preceding case, is opposed to the force towards AB, and diminishes it by the quantity Ag. Thus, in the first case, the moveable body must traverse more space, since its speed is increased, and for the contrary reason, it must traverse less in the second case, for the time of its motion is assumed to be the same.

§280. Since two sides of a triangle taken together are always longer than the third (Euclid, Book I, Prop. 20), the Body A goes along a shorter path when it obeys any two powers at the same time, than if it obeyed each of them individually in succession.

§281. It is seen by inspection of Fig. 24 that the path of a moveable body can be the diagonal of an infinity of diverse parallelograms; for, the line AD is at the same the diagonal of parallelograms AEBD, AfDh, etc.

§282. Thus, a Body can traverse the same straight line in the same time, whether it is impelled by several forces or by a single force. The Body A, for example, will equally traverse the line AD in a given time if it is impelled by a single force directed toward AD and that impresses upon it this speed AD, or by the two forces AB and AE, which impress upon it the speeds designated by these lines AB and AE. **{Fig. 23}** One can likewise consider the Body that traverses the line AD as being moved by these two differerent forces, or by a single [force] that is equal to them; for the speed or the motion toward AD contains only the speed AB in the direction AB and the speed AE in the direction AE. Thus, the effect is always the same, whether the moveable body is impelled by three or four, or any number whatsoever of combined forces, or indeed by a single force that impresses on it the same speed

in the same direction as that in which the action of these different forces would combine. And one can likewise consider all these forces as being combined in that [force] which represents them, or this single force as being divided into the forces that compose it.

§283. {On the resolution and composition of motion. Utility of this method.}
These two different ways of considering the motion of Bodies are called resolution and composition.

This method is of great use, and of great utility in Mechanics, for discovering the quantity of the action of Bodies that act obliquely on one another.

§284. {How one knows the path of the moveable body in all compositions of motion} One knows the path of a moveable body moved by any two forces, when one knows the speed that each of these two forces impresses upon it and the angle that their directions make between them; for this path is the third side of a triangle of which one knows the two other sides and the angle between them.

§285. By this means one knows the path of a Body that obeys any number whatsoever of forces that act upon it at the same time. For when one has determined the path that two of these forces make the moveable body traverse, by the rule of the preceding paragraph, this path becomes the side of a new triangle, in which the line representing the third force becomes the second side, and the path of the moveable body the base; by proceeding likewise up to the last force, one will attain knowledge of the path of the moveable body that arises from the combined action of all the forces acting upon it. **{Fig. 27}** For, the body A impelled by the two forces E and D in the directions and with the speeds AB and AG will describe the diagonal AH, and impelled then by the force C in the direction and with the speed AF, will traverse the line AT. Finally, the force M will make it describe the line AL by impressing upon it the direction and the speed AK. Thus, AL is the path of the moveable body A, impelled at the same time by the forces: E, D, C, and M.

§286. A Body can undergo several motions at the same time; for a Body that we throw horizontally in a boat undergoes the projectile motion that we communicate to it, and that which heaviness impresses upon it at each moment toward the Earth; it participates additionally in the motion of the vessel in which it is. The River on which this vessel is flows unceasingly, and the Body participates in this motion. The Earth on which this river flows turns on its axis in twenty-four hours: yet another motion that the body shares. Finally, the Earth has its annual motion around the Sun, the revolution of its poles, the precession [balancement] of its equator, etc., and the Body under consideration participates in all these motions. But only the first two belong to it, in relation to those who are transported with the Body on the boat;[4] for, all Bodies which have a motion in common with us are as at rest in relation to us.

§287. A Body that receives several determinations remains in the last as in the last degree of speed if it

[4] Or: "But only the first two belong to it in relation to those who are transported with the Body on the boat". Whether or not the comma should be there depends on her theory of motion, and we don't (yet) know enough to decide.

is left to itself and if no force acts any longer upon it. It conserves this determination and this speed until it encounters some obstacle that makes it lose its motion, by consuming its force, or if some new force changes its direction This effect is a necessary consequence of the first Law of motion, founded on the force of inertia of matter.

§288 Composite motion can be uniformly or non-uniformly accelerated, just like simple motion. **{On motion in a curved line}.**

If the two forces that impel the Body are unequally accelerated, or indeed if one of the motions is accelerated whilst the other is uniform, the line described by the Body in motion will no longer be a right line, but a curved line, the curvature of which will be different according to the combination of the inequalities of the forces that make the Body describe it; for this Body will obey each of the forces that impel it according to the quantity of each force's action on it (2^{nd} Law §229). Thus, for example, if there is one of these forces that renews its action at each instant, while the action of the other force remains the same, the path of the moveable body will be changed at every moment; and it is in this way that all thrown Bodies fall back toward the earth (Chap. 19).

§289. **{Motion in a curved line is always a composite motion}** All motion in a curved line is necessarily a motion composed from the motion that makes the Body go in a right line and the motion that draws it away; for to describe a curved line just is to change direction at every moment.

§290. {Motion is always in a right line in an infinitely small instant} Motion always takes place in a right line; for although a Body moved by two forces that impress unequally accelerated speeds upon it describes a curved line, nevertheless the partial motion of this Body is always in a right line, and its total motion is in a curved line only because the points toward which the moveable body is directed change at each moment, and the minuteness of the right lines that this movable body traverses at each instant prevent us from distinguishing each one individually, so that all this assemblage of lines that are infinitely small and inclined to one another appears to us a single curved line. But each of these little right lines represents the direction of motion at each infinitely small instant, and this is the diagonal of a parallelogram formed from the direction of the actual forces that act upon this Body. Thus, motion is always in a right line at each infinitely small instant, just as it is always uniform.

§291. If the accelerative force suddenly ceased to act, the Body would continue to move in the right line along which it would find itself directed at that instant; for every Body moves continues to move in a right line and with equal speeds as long as nothing hinders it according to the first Law of motion (§229). It is in following this law that every Body that moves in a circle tends to escape along its tangent; and this is what we call *centrifugal force*.

§292. There is yet another type of circular motion. This is the relative motion of a Body that rotates on itself, as the Earth, for example, in its diurnal motion: this is made, then, by the parts of this Body that

tend to describe the infinitely small straight lines of which I have just spoken (§290).

One can define this type of circular motion: *a motion in which the parts change place, although the whole does not*.